北京航空航天大學
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SCHOOL OF ECONOMICS AND MANAGEMENT

Generalized Linear Models

Lecture 9: Mixed Effect Models for non-Gaussian Responses

GLM

Generalized Linear Mixed Models

- Combine GLMs with random effects
- y_i from exponential family distribution $f(y_i, \theta_i, \phi)$
- $E(y_i) = \mu_i$
- Link function g: $g(\mu_i) = \boldsymbol{x}_i'\beta + \boldsymbol{z}_i'\gamma$
- β are fixed effects; γ are random effects.
- $\gamma \sim N(\mathbf{0}, \boldsymbol{D})$ with density $h(\gamma|\boldsymbol{D})$

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Likelihood

$$L(\beta, \phi, \mathbf{D}) = \prod_{i=1}^{n} \int f(y_i|\beta, \phi, \gamma) h(\gamma|\mathbf{D})$$

- Can only solve integrals if f and h both normal

Numerical integration of likelihood

Likelihood

$$L(\beta, \phi, \boldsymbol{D}) = \prod_{i=1}^{n} \int f(y_i | \beta, \phi, \gamma) h(\gamma | \boldsymbol{D})$$

- Use numerical integration to approximate integrals
- More accurate than PQL
- Can be slow or impossible for complex models
- Inference will be problematic, as for MLE with LMMs

Penalized Quasi Likelihood

1. Transform fitted values:

$$\eta_i = g(\mu_i) = \mathbf{x}_i'\beta + \mathbf{z}_i'\gamma$$

2. Create pseudo-responses:

$$\tilde{y}_i^j = \hat{\eta}_i^j + (y_i - \hat{\mu}_i^j)\left.\frac{d\eta}{d\mu}\right|_{\hat{\eta}_i^j}$$

where j is iteration in optimization algorithm

3. Find $V(\tilde{y}_i|\gamma)$
4. Use weighted linear mixed effects models

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- fast
- approximate inference
- biased estimates, esp. for binary data or low counts
- even worse inference than regular LMM

Bayesian methods

- Much more accurate inference
- Allow for prior information and flexibility
- Usually take more computation
- Inferential form different
- Require additional software (either INLA or STAN).